

# Mutually Exclusive and Overlapping Events

**GOAL** Find the probability of mutually exclusive, overlapping, and complementary events.

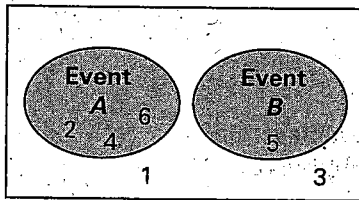
When you consider the probability of two events occurring, the events are called **compound events**. *Mutually exclusive* and *overlapping* events are examples of compound events.

**Mutually exclusive events** are events that have no outcomes in common.

**Overlapping events** are events that have one or more outcomes in common. The Venn diagrams below show how the events that involve rolling a number cube are related.

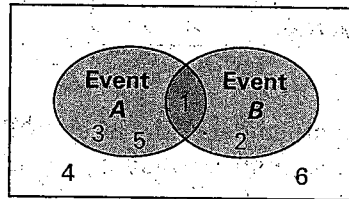
### Mutually Exclusive Events

Event A: Get an even number.  
Event B: Get a 5.



### Overlapping Events

Event A: Get an odd number.  
Event B: Get a number less than 3.



Use the formula below to find the probability of mutually exclusive events.

### Probability of Mutually Exclusive Events

For two mutually exclusive events  $A$  and  $B$ , the probability that either of the events occurs is the sum of the probabilities of the events.

$$P(A \text{ or } B) = P(A) + P(B) \quad \text{Events } A \text{ and } B \text{ are mutually exclusive.}$$

### EXAMPLE 1 Probability of Mutually Exclusive Events

The table below lists the number of each type of fish in a fish tank. What is the probability that a randomly chosen fish is a minnow or a rainbowfish?

Type	angelfish	goldfish	minnow	rainbowfish
Number	12	9	5	4

#### SOLUTION

Find the probability of each event. Then add the probabilities.

$$P(\text{minnow}) = \frac{5}{30} \quad \text{Of the 30 fish, 5 are minnows.}$$

$$P(\text{rainbowfish}) = \frac{4}{30} \quad \text{Of the 30 fish, 4 are rainbowfish.}$$

$$P(\text{minnow or rainbowfish}) = \frac{5}{30} + \frac{4}{30} = \frac{9}{30} = 0.3$$

The probability that the fish is a minnow or a rainbowfish is 30%.

**Complementary Events** Two mutually exclusive events in which one or the other event *must* occur are called **complementary events**.

### Probability of Complementary Events

The sum of the probabilities of two complementary events is 1. If  $A$  and  $B$  are complementary events, then  $P(A) + P(B) = 1$ .

The above formula means that for an event  $A$  and its complement, which may be written as "not  $A$ ,"  $P(A) + P(\text{not } A) = 1$  and  $P(\text{not } A) = 1 - P(A)$ .

### EXAMPLE 3 Using Complementary Events

There is an 80% chance that it will rain tomorrow. What is the probability that it will not rain tomorrow?

#### SOLUTION

The event that it rains tomorrow and the event that it does not rain tomorrow are complementary events.

$$\begin{aligned} P(\text{not rain}) &= 1 - P(\text{rain}) \\ &= 1 - 0.8 = 0.2 \end{aligned}$$

The probability that it will not rain tomorrow is 20%.

#### CHECK Example 3

3. The probability that your raffle ticket is not a winner is 0.9998. What is the probability that your raffle ticket is a winner?

### EXERCISES

Tell whether the events are *mutually exclusive* or *overlapping*.

1. You randomly choose a letter from the word MISSISSIPPI.

**Event A:** You get an S.

**Event B:** You get an I.

2. You conduct a survey about pet ownership.

**Event A:** The respondent is a cat owner.

**Event B:** The respondent is a dog owner.

3. You randomly choose a student in your class.

**Event A:** The student plays baseball.

**Event B:** The student plays basketball.

4. You randomly choose a member of your school band.

**Event A:** The student plays the clarinet.

**Event B:** The student does not play the clarinet.

## Probability of Overlapping Events

For two overlapping events  $A$  and  $B$ , the probability that either of the events occurs is the sum of the probabilities of the events, minus the probability that both events occur.

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) \quad \text{Events } A \text{ and } B \text{ are overlapping.}$$

### EXAMPLE 2 Probability of Overlapping Events

A high school has four soccer teams. The number of players on each team is shown in the table. A soccer player is randomly chosen. What is the probability that the player is a female student or a player on a varsity soccer team?

	Girls	Boys
Junior Varsity	19	17
Varsity	23	21

#### SOLUTION

Let event  $A$  be “choose a female student” and let event  $B$  be “choose a player on a varsity soccer team.” The events are overlapping, so find the probabilities of event  $A$ , event  $B$ , and events  $A$  and  $B$ .

$$P(A) = \frac{19 + 23}{19 + 23 + 17 + 21} = \frac{42}{80} \quad \text{Of the 80 players, 42 are female.}$$

$$P(B) = \frac{23 + 21}{19 + 23 + 17 + 21} = \frac{44}{80} \quad \text{Of the 80 players, 44 are on varsity.}$$

$$P(A \text{ and } B) = \frac{23}{80} \quad \text{Of the 80 players, 23 are on girls' varsity.}$$

Then find  $P(A \text{ or } B)$ .

$$P(A \text{ or } B) = \frac{42}{80} + \frac{44}{80} - \frac{23}{80} = \frac{63}{80} \approx 0.788$$

The probability that the player is female or on varsity is about 78.8%.

#### CHECK Examples 1 and 2

Tell whether the situation describes *mutually exclusive* or *overlapping events*. Then answer the question.

1. A gumball machine contains 25 red, 18 orange, and 17 yellow gumballs. What is the probability that a randomly chosen gumball is orange or yellow?
2. Each whole number from 1 through 10 is written on a separate piece of paper and placed in a bag. What is the probability that a randomly chosen number is an even number or a number greater than 5?

Events  $A$  and  $B$  are mutually exclusive. Find the missing probability.

5.  $P(A) = 0.62$

$P(B) = 0.1$

$P(A \text{ or } B) = \underline{\quad?}$

6.  $P(A) = 0.5$

$P(B) = \underline{\quad?}$

$P(A \text{ or } B) = 0.9$

7.  $P(A) = \underline{\quad?}$

$P(B) = 12.5\%$

$P(A \text{ or } B) = 80\%$

Events  $A$  and  $B$  are overlapping. Find the missing probability.

8.  $P(A) = 0.375$

$P(B) = 0.5$

$P(A \text{ and } B) = 0.3$

$P(A \text{ or } B) = \underline{\quad?}$

9.  $P(A) = 0.64$

$P(B) = 0.24$

$P(A \text{ and } B) = \underline{\quad?}$

$P(A \text{ or } B) = 0.6$

10.  $P(A) = \underline{\quad?}$

$P(B) = 16\%$

$P(A \text{ and } B) = 10\%$

$P(A \text{ or } B) = 25\%$

Tell whether the situation describes *mutually exclusive* or *overlapping events*. Then answer the question.

11. You roll a number cube. What is the probability that you roll an even number or a number less than 5?
12. You roll a number cube. What is the probability that you roll an odd number or a 6?
13. An aquarium has 38 African penguins (21 males and 17 females) and 15 rockhopper penguins (6 males and 9 females). What is the probability that a randomly chosen penguin is a male or a rockhopper penguin?
14. Of the pre-owned vehicles at a dealership, 9 are one year old, 7 are two years old, 4 are three years old, and the remaining 10 are four years old or older. What is the probability that a randomly chosen pre-owned vehicle is one or two years old?

In Exercises 15–17, use the following information.

A bag contains red, blue, and yellow marbles. The probability of choosing a red marble is 0.35. The probability of choosing a blue marble is 0.2.

15. What is the probability of not choosing a red marble?
16. What is the probability of not choosing a blue marble?
17. What is the probability of choosing a yellow marble?

In Exercises 18–20, use the following information.

The word *or* is represented in set notation by the union ( $\cup$ ) of sets. This is because an outcome is in event  $A$  or event  $B$  if and only if the outcome is in the union of the sets,  $A \cup B$ .

18. What is  $P(A \cup C)$  for the events at the right?
19. What is  $P(B \cup C)$  for the events at the right?
20. What is  $P(A \cup B)$  for the events at the right? Explain why your result makes sense.

